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by G. V. Druzhinin

-USSR-

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PREDICTION OF DURABILITY OF ELEMENTS AND AUTOMATIC SYSTEMS WITH VECTORIAL DETERMINING PARAMETERS

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Durability means the ability of elements and systems to remain in perfect order during storage or operation. Statistical studies of durability deal with random change processes in determining element parameters as a result of wear or deterioration (references 1,3). The cited papers assume that each element is characterized by a single determining parameter. There are cases, however, when a system or element is characterized by several determining parameters (a vectorial determining parameter). It thus becomes necessary to deal with vectorial random processes of change in the determining parameters and statistical bounds of elements. The purpose of the present article is to predict systems and element durability on the basis of the state of elements at the present time.

1. Characteristics of vectorial determining parameters and statistical bounds of elements. Any study of durability is almost always connected with the extrapolation of determining parameters and statistical bounds. At the same time, the possibility of measuring determining parameter and statistical bound values is extremely limited. This makes it necessary to employ approximate methods of

predicting element and systems durability. It is very convenient to approximate vectorial determining parameters by means of linear vectorial random time functions with components of the form

$$H_\nu(t) = A_\nu + B_\nu t \qquad (\nu = 1, 2, \ldots) \tag{1.1}$$

Here A_ν is the random initial value of the ν-th component $H_\nu(t)$ of the vectorial random function H(t); B_ν is the random mean (with respect to time) rate of change of the ν-th component of H(t).

The formulas giving the mathematical expectations, dispersions, and correlative functions of the vectorial linear random function H(t) components are analogous to the corresponding formulas for the scalar linear random function (reference 1). The mutual correlative function of the vectorial linear random function has the form

$$K_{\eta_\nu \eta_\mu}(t, t') = k_{a_\nu a_\mu} + (t + t') k_{a_\nu b_\mu} + tt' k_{b_\nu b_\mu} \tag{1.2}$$

Setting $\nu = \mu$ in formula (1.2), we obtain the formula for the correlative function of the μ-th component of H(t), while for $\nu = \mu$ and $t = t'$ we have the formula for the dispersion of this component.

Making use of linear vectorial random functions, it is possible to characterize the vectorial determining parameters with a relatively small number of constant numerical characteristics; this simplifies considerably the study of element and systems durability. In addition to this, the use of the indicated functions makes it possible to extrapolate the random change processes in the determining parameters and statistical bounds of elements on the basis of the minimum number of cross-sections for these processes (two). In calculating the characteristics of linear random functions on the basis of the numerical characteristics of two cross-sections of these functions, each realization is replaced with a rectilinear secant; there is a statistical linearization of the actual random process.

The normally distributed vectorial linear random function is fully characterized by the following constant numerical characteristics: the mathematical expectations for the initial values of the components m_{a_1}, m_{a_2}, ...; the mathematical expectations of the rates of change of the components m_{b_1}, m_{b_2}, ... and the central second-order

moments

$$k_{a_\nu a_\mu},\ k_{a_\nu b_\mu},\ k_{b_\nu b_\mu} \quad \text{for } \nu, \mu = 1, 2, \ldots, h$$

The introduction of additional assumptions makes possible a considerable reduction in the number of numerical characteristics describing the linear vectorial random function H(t).

Using theorems on the numerical characteristics of random quantities, it is possible to derive formulas for finding the numerical characteristics of a linear vectorial random function on the basis of the numerical characteristics of two cross-sections of this function. The mathematical expectations of the initial values and rates of change of vectorial linear random function H(t) components are expressed by the formulas

$$m_{a_\nu} = \frac{t_{i+1} m_{\eta_{\nu i}} - t_i m_{\eta_{\nu(i+1)}}}{t_{i+1} - t_i} \qquad (1.3)$$

$$m_{b_\nu} = \frac{m_{\eta_{\nu(i+1)}} - m_{\eta_{\nu i}}}{t_{i+1} - t_i} \qquad (1.4)$$

In formulas (1,3) and (1.4) $m_{\eta_{\nu i}}$ is the mathematical expectation of the ν-th component ($\nu = 1, 2, \ldots, h$) of the vectorial linear random function H(t) in the i-th cross-section.

The coupling moments of the initial values

$$k_{a_\nu a_\mu} = \frac{1}{(t_{i+1} - t_i)^2} \{ t_{i+1}^2 k_{\eta_{\nu i}\eta_{\mu i}} - t_i t_{i+1} (k_{\eta_{\nu(i+1)}\eta_{\mu i}} + k_{\eta_{\nu i}\eta_{\mu(i+1)}}) + t_i^2 k_{\eta_{\nu(i+1)}\eta_{\mu(i+1)}} \} \quad (\nu, \mu = 1, 2, \ldots, h) \qquad (1.5)$$

In formula (1.5) $k_{\eta_{\nu(i+1)}\eta_{\mu i}}$ is is the coupling moment of cross-section t_{i+1} of the ν-th component of the linear random function H(t) and the cross-section t_i of the μ-th component of the same random function. The rest of the definitions are read off accordingly. If in formula (1.5) we set $\nu = \mu$, we have a formula for the dispersion of the initial value of the -th component of the linear vectorial random function H(t).

The coupling moments of the rates of change of the vectorial linear random function components are

$$k_{b_\nu b_\mu} = \frac{1}{(t_{i+1}-t_i)^2}\left[k_{\eta_{\nu(i+1)}\eta_{\mu(i+1)}} - k_{\eta_{\nu(i+1)}\eta_{\mu i}} - k_{\eta_{\nu i}\eta_{\mu(i+1)}} + k_{\eta_{\nu i}\eta_{\mu i}}\right] \quad (\nu, \mu = 1, 2, \ldots, h) \tag{1.6}$$

For $\nu = \mu$ we obtain a formula for the dispersion $\sigma_{b_\nu}^2 = k_{b_\nu b_\nu}$ of the ν-th component.

The coupling moments of the initial values of the ν-th components of the linear vectorial random function H(t) with the rates of change of the μ-th components of this function can be determined from the formula

$$k_{a_\nu b_\mu} = \frac{1}{(t_{i+1}-t_i)^2}\left[t_{i+1}k_{\eta_{\nu i}\eta_{\mu(i+1)}} - t_i k_{\eta_{\nu(i+1)}\eta_{\mu(i+1)}} - t_{i+1}k_{\eta_{\nu i}\eta_{\mu i}} + t_i k_{\eta_{\nu(i+1)}\eta_{\mu i}}\right] \quad (\nu, \mu = 1, 2, \ldots, h) \tag{1.7}$$

2. The calculation of the durability of elements with vectorial determining parameters. At a moment of time t_i an element is characterized by a system of random quantities $H_{1i}, H_{2i}, \ldots, H_{hi}$. Let us denote the mutually independent critical parameter values, at which the element is considered out of order (permissible limits) by $\omega_1, \omega_2, \ldots, \omega_h$. Then the probability G_i that the element will be in working order at a moment of time t_i is equal to the probability of the simultaneous fulfillment of the inequalities $H_{1i} > \omega_1$, $H_{2i} > \omega_2, \ldots, H_{hi} > \omega_h$, i.e.,

$$G_i = \int_{\omega_1}^{\infty}\int_{\omega_2}^{\infty}\cdots\int_{\omega_h}^{\infty} f_i(\eta_1, \eta_2, \ldots, \eta_h)\, d\eta_1 d\eta_2 \ldots d\eta_h \tag{2.1}$$

where $f_i(\eta_1, \eta_2, \ldots, \eta)$ is the probability density of the random vector H_i with components $H_{1i}, H_{2i}, \ldots, H_{hi}$.

In practice it is difficult to expect that the vectorial determining parameter of an element will have more than three components. This fact is engendered by mathematical difficulties and the fact that elements are usually quite adequately characterized by a vectorial random function with a small number of components.

In a number of practical problems the components of a normally distributed vectorial random process H(t) are so

weakly correlated that they can be considered uncoupled and consequently independent. Then integral (2.1) disintegrates into the integral product

$$G_i = G_1 G_2 \dots G_h = \int_{\omega_1}^{\infty} f_{1i}(\eta_1)\, d\eta_1 \int_{\omega_2}^{\infty} f_{2i}(\eta_2) d\eta_2 \dots \int_{\omega_h}^{\infty} f_{hi}(\eta_h)\, d\eta_h \quad (2.2)$$

where $f_{1i}(\eta_1), \dots, f_{hi}(\eta_h)$ are the normal distribution laws for the components of the random vector H_i

$$f_{\nu i}(\eta_\nu) = \frac{1}{\sigma_{\eta_{\nu i}}\sqrt{2\pi}} \exp\left[-\frac{(\eta - m_{\eta_{\nu i}})^2}{2\sigma_{\eta_{\nu i}}^2}\right] \quad (2.3)$$

In formula (2.3), $m_{\eta_{\nu i}}$ is the mean value of the ν-th determining parameter of an element at moment of time t_i; $\sigma_{\eta_{\nu i}}$ is the mean-square deviation of the ν-th determining parameter at moment of time t_i.

With normal distribution laws $f_{\nu i}(\eta_\nu)$, formula (2.2) can be written in the form

$$G_i = \left[\tfrac{1}{2} - \Phi(u_{1i})\right] \dots \left[\tfrac{1}{2} - \Phi(u_{hi})\right] \quad (2.4)$$

where $$u_{\nu i} = \frac{\omega_\nu - m_{\eta_{\nu i}}}{\sigma_{\eta_{\nu i}}}$$

while $$\Phi(u_{\nu i}) = \frac{1}{\sqrt{2\pi}} \int_0^{u_{\nu i}} \exp\left[-\frac{x^2}{2}\right] dx$$ is a Gaussian function.

The probability density for the duration that the element is in working order $f_\omega(t)$ is the rate of change of probability $1-G_i$ that the element is out of order

$$f_\omega(t)\big|_{t=t_i} = -\frac{dG_i}{dt}\bigg|_{t=t_i} \quad (2.6)$$

For a vectorial determining parameter with independent components

$$f_\omega(t)\big|_{t=t_i} = \left[\frac{dG_1}{dt} G_2 \dots G_h + \frac{dG_2}{dt} G_1 G_3 \dots G_h + \frac{dG_h}{dt} G_1 G_2 \dots G_{h-1}\right]_{t=t_i} \quad (2.7)$$

Since it is usually the case that for elements with rather short terms of service the values of $G_1, \dots, G_h$

differ little from unity,

$$f(t)\big|_{t=t_i} \approx -\sum_{\nu=1}^{h} \frac{dG_\nu}{dt}\bigg|_{t=t_i} \tag{2.8}$$

Using formula (2.8) it is possible, in principle, to obtain an analytic expression for the distribution law for the time that the element is in good working order. In the general case, however, this expression would be extremely unwieldy. In practice, the density of the probability of the time that the element is in good working order is conveniently calculated by means of the mean value f_i on each of the intervals (t_i, t_{i+1}). In accordance with (2.8)

$$f_i \approx \sum_{\nu=1}^{h} \frac{G_{\nu i} - G_{\nu(i+1)}}{t_{i+1} - t_i} \tag{2.9}$$

For the normally distributed vectorial determining parameter, formula (2.9) is conveniently expressed by means of the Gaussian function

$$f_i \approx \sum_{\nu=1}^{h} \frac{\Phi(u_{\nu(i+1)}) - \Phi(u_{\nu i})}{t_{i+1} - t_i} \tag{2.10}$$

where u_ν is determined in accordance with (2.5).

Thus, the approximate prediction on the basis of a minimum number of element parameter measurements of the distribution law $f(t)$ for the time that the element is in working order for an element with a normally distributed vectorial determining vector having independent components, it is necessary:

a) to use measurements of the realizations of the vectorial determining parameter components of similar elements at two moments of time t_i and t_{i+1} to calculate the statistical numerical characteristics: mean values, dispersions, and correlative cross-section moments;

b) using the resultant data to find by means of formulas (1.3)-(1.7) the characteristics $m_{\eta_\nu}(t)$ and $\sigma_{\eta_\nu}(t)$ of the vectorial linear random function $H(t)$;

c) to make use of formulas (2.5) and (2.10) to find the values of f_i for various time intervals (t_i, t_{i+1}). The calculated values of f_i are employed to construct a

histogram which is smoothed out by a continuous curve.

3) <u>The calculation of the durability of systems with vectorial element and systems determining parameters.</u> A sufficiently simple solution to the problem amenable to use in various engineering applications can be obtained only in the case of independent vectorial determining parameter components for elements and systems. It is then possible to use ideas suggested in (reference 3) for scalar determining parameters.

In systems with a relatively small number of elements, it is convenient to look upon the vectorial determining parameter of a system $W(t)$ as a function of vectorial determining parameters of elements $H_1(t), \ldots, H_n(t)$. At a fixed moment of time t_i the random vector W_i is a function of n random vectors $H_{\ell i}$, each of which has several components. The linearization of the dependence

$$W_{ri} = \varphi_r(H_{11i}, \ldots, H_{1ai}; H_{21i}, \ldots, H_{2bi}; \ldots; H_{n1i}, \ldots, H_{nzi} \quad (3.1)$$

by expansion in a Taylor series and the elimination of all terms of higher than the first order, we find the approximate values for the mathematical expectations of vector W_i components by means of the formula

$$m_{w_{ri}} = \varphi_r(m_{\eta_{11i}}, \ldots, m_{\eta_{1ai}}; m_{\eta_{21i}}, \ldots, m_{\eta_{2bi}}; \ldots; m_{\eta_{n1i}}, \ldots, m_{\eta_{nzi}}) \quad (3.2)$$

In formulas (3.1) and (3.2) a, b, ..., z is the number of components of each of the random vectors $H_{1i}, H_{2i}, \ldots, H_{ni}$; the components of the random vector W_i are indicated by the index $r = 1, 2, \ldots, m$.

With uncorrelated vectorial random processes $H_\ell(t)$ $(\ell = 1, 2, \ldots, n)$ having uncorrelated components, the dispersions of the random vector W_i components will be

$$\sigma_{w_{ri}}^2 = \sum_{l=1}^{n} \sum_{\nu=1}^{a_\nu} \left(\frac{\partial \varphi_r}{\partial \eta_{l\nu i}}\right)_{m_{\eta_{l\nu i}}} \sigma_{\eta_{l\nu i}}^2 \quad (3.3)$$

In formula (3.3) $\sigma^2_{\eta_{\ell\nu i}}$ is the dispersion of the ν-th component of the random vector $H_{\ell i}$; a_ℓ is the number of components of each of the random vectors $H_{\ell i}$. When $H_{1i}, \ldots, H_{ni}$ are scalar random quatities, formula (3.3) takes the form

$$\sigma_{w_{ri}}^2 = \sum_{l=1}^{n} \left(\frac{\partial \varphi_r}{\partial \eta_{li}}\right)_{m_{\eta_{li}}} \sigma_{\eta_{li}}^2 \quad (3.4)$$

Having calculated the values of m_{w_r} and $\sigma^2_{w_r}$ for two moments of time t_i and t_{i+1}, it is possible to use the formulas given in the last section to find the mean probability density for the time during shich the element is in working order over the interval (t_i, t_{i+1}).

Assuming that the independent components of the vectorial linear random function $W(t)$ are uniform and have mean-square deviations

$$\sigma_{w_r} = \frac{1}{2} \left(\sigma_{w_{ri}} + \sigma_{w_{r(i+1)}}\right) = \text{const}$$

it is possible to find the entire distribution law for the time the system is in working order in a rough approximation. A more precise calculation of the law of distribution $f(t)$ requires that we find the characteristics $m_{w_r}(t)$ and $\sigma_{w_r}(t)$ of the components of the vectorial linear random function $W(t)$. To calculate these characteristics by means of the formulas found in reference 1, it is necessary to know the coupling moment of the random quantities W_{ri} and $W_{r(i+1)}$. The formula for the latter may be obtained by substituting into the correlative moment formula values of the dependence (3,1) linearized by expansion in a Taylor series. After the transformations we have

$$k_{w_{ri} w_{r(i+1)}} = \sum_{l=1}^{n} \sum_{\nu=1}^{n_\nu} \left(\frac{\partial \varphi_r}{\partial \eta_{\nu l}}\right)_{m_{\eta_{\nu l i}}} \left(\frac{\partial \varphi_r}{\partial \eta_{\nu l}}\right)_{m_{\eta_{\nu l (i+1)}}} k_{\eta_{\nu l i} \eta_{\nu l (i+1)}} \qquad (3.5)$$

In formula (3.5) $k_{\eta_{\nu l i} \eta_{\nu l (i+1)}}$ is the coupling moment of the values for the ν-th component of the determining parameter of the l-th element in the i-th and (i+1)-th cross-sections.

The calculation of W_{ri} and $W_{r(i+1)}$ with essentially non-linear dependences (3,1) can be carried out by methods described in reference 3.

The calculation of the durability of a complex system according to statistical element boundaries with vectorial determining parameters is in many ways analogous to the corresponding calculation with scalar element determining parameters (reference 3). The difference consists in the fact that the case under consideration involves the random vector Z_i related to moment of time t_i with components

$$Z_{\nu i} = H_{\nu i} - \Omega_{\nu i} \qquad (3.6)$$

In formula (3.6) $H_{\nu i}$ is the value of the ν-th component of the determining parameter of the element at time t_i, and $\Omega_{\nu i}$ is the value of the same component of the vectorial statistical bound of this element. With normal distribution laws and uncorrelated random vector components H_i and Ω_i, the mean probability density of the time during which the element is in working order over the period (t_i, t_{i+1}), in accordance with formulas (2.10) and (3.6), will be

$$f_i \approx \sum_{\nu=1}^{k} \frac{\Phi(u_{\nu(i+1)}) - \Phi(u_{\nu i})}{t_{i+1} - t_i} \qquad (3.7)$$

where $\Phi(u_{\nu i})$ is a Gaussian function and

$$u_{\nu i} = \frac{m_{\eta_{\nu i}} - m_{\omega_{\nu i}}}{\sqrt{\sigma^2_{\eta_{\nu i}} + \sigma^2_{\omega_{\nu i}}}} \qquad (3.8)$$

In formulas (3.7) and (3.8), $m_{\eta_{\nu i}}$ and $m_{\omega_{\nu i}}$ are the mathematical expectations of the values of the ν-th components of the determining parameter and the statistical bound at time t_i; $\sigma_{\eta_{\nu i}}$ and $\sigma_{\omega_{\nu i}}$ are the mean-square deviations of these quantities from their mathematical expectations.

In order to find the distribution law f(t) for the time during which the element is in working order, it is necessary to determine from formulas (1.3)-(1.7) the numerical characteristics which determine the dependences $m_{\eta}(t)$, $m_{\omega}(t)$, $\sigma_{\eta}(t)$, and $\sigma_{\omega}(t)$. Then formulas (3.7) and (3.8) are used to find the f_i values for each interval which are smoothed out by a continuous curve.

Taking into consideration correlations between the components of random vectors H_i and Ω_i considerably complicates the process of calculating systems durability on the basis of statistical element bounds with vectorial determining parameters.

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